

Co-design of aperiodic sampled-data min-jumping rules for linear impulsive, switched impulsive and sampled-data systems

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Abstract

An aperiodic sampled-data min-jumping rule is proposed for linear impulsive systems, a class of systems encompassing switched and sampled-data systems as particular cases. Several sufficient co-design conditions characterizing the stabilization of a given linear impulsive system subject to the considered min-jumping rule and a sampled-data state-feedback controller are formulated in terms of discrete-time Lyapunov-Metzler conditions. These conditions are then exactly reformulated as clock-dependent conditions known to be more convenient to verify and that allow for an immediate extension of the results to uncertain systems and to performance analysis. Several examples pertaining on sampled-data control of switched and impulsive systems are given for illustration.

1 Introduction

The objective of the paper is the development of methods for the co-design of jump-rules and state-feedback controllers for linear impulsive systems, a general class of hybrid systems that encompasses switched and sampled-data systems as particular instances [1, 2]. The approaches are based on a so-called *min-jumping rule*, which is analogous to the min-switching rule considered in [3, 4] in the context of switched systems. The underlying idea of this switching rule is to consider multiple Lyapunov functions and select the mode that minimizes the value of the corresponding Lyapunov function at any time. Another approach [5] is based on the use of a common Lyapunov function and the switching rule is chosen such that the derivative of the Lyapunov function is minimum at any time. Although slightly different, these approaches share the underlying assumption that the state/output of the system is continuously measured, which may be impractical from an implementation point of view or may lead to undesired chattering. A solution to the chattering problem that may arise in [3] is proposed in [6] where a refractory period for the switching signal is imposed in the form of a minimum dwell-time constraint.

An alternative method where state measurements are made at certain time instants and used to update the control input accordingly is proposed here. Such a scenario occurs, for instance, in networked control systems where actuation decisions are made upon reception of measurements from sensors, which may happen aperiodically due to the presence of sampling, jitter, delays and packet loss [7, 8]. The advantage of such an approach is twofold: it rules out the chattering phenomenon of the continuous-time methods and is realistic from an implementation viewpoint since no continuous measurement is assumed. It is assumed here that the measurements from sensors arrive at discrete time instants which are assumed to satisfy a mild range dwell-time condition [2, 9]. Hence, both periodic and aperiodic measurements are considered and can be easily defined in a way to incorporate jitter, small delays and self/event-triggered sampling mechanisms [10]. The proposed approach is then applied to impulsive systems and switched-impulsive systems which can be used to model networked control systems subject to delays and communication outages, systems controlled by multiple controllers or systems with limited actuation resources [7]. To the author's knowledge, this is the first time that convex conditions for the co-design of asynchronous switching/jumping rules and a sampled-data state-feedback controller are obtained for impulsive and switched impulsive systems.

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Sufficient stabilization conditions are first stated as discrete-time Lyapunov-Metzler conditions, which are then reformulated into equivalent clock-dependent conditions [2, 11]. These conditions have been shown to be more suitable for computational purposes and to be easily generalizable to uncertain systems and to performance characterization such the L_2 -performance; see e.g; [6, 12]. The main drawback is that these conditions are infinite-dimensional and need to be relaxed using, for instance, methods based on sum of squares [2, 11, 13, 14].

Outline. The paper is structured as follows. The main results for impulsive systems are developed in Section 2 and are extended to switched systems in Section 3. Some illustrative examples are finally discussed in Section 4.

Notations. The set of symmetric matrices of dimension n is denoted by \mathbb{S}^n and for $A, B \in \mathbb{S}^n$, $A \preceq B$ means that $A - B$ is negative semidefinite. The cone of symmetric positive (semi)definite matrices of dimension n is denoted by $(\mathbb{S}_{\succeq 0}^n \ \mathbb{S}_{> 0}^n)$. The n -dimensional vector of ones is denoted by $\mathbb{1}_n$.

2 Results for impulsive systems

Let us consider in this section the following impulsive system with multiple jump maps and sampled-data state-feedback:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), t \in \mathbb{R}_{\geq 0} \setminus \{t_k\}_{k=0}^{\infty} \\ u(t) &= K_{\sigma(t_k^+)}^1 x(t_k) + K_{\sigma(t_k^+)}^2 u(t_k), t \in (t_k, t_{k+1}] \\ x(t_k^+) &= J_{\sigma(t_k^+)} x(t_k), k \in \mathbb{Z}_{\geq 0} \end{aligned} \quad (1)$$

where $x(\cdot), x_0 \in \mathbb{R}^n$, $u(\cdot) \in \mathbb{R}^m$ are the state of the system, the initial condition and the control input. The notation $x(t_k^+)$ is defined as $x(t_k^+) := \lim_{s \downarrow t_k} x(s)$ and we have that $x(t_k) = \lim_{s \uparrow t_k} x(s)$, i.e. the trajectories are left-continuous. The signal $\sigma(t) \in \{1, \dots, N\}$ is assumed to be piecewise constant and to only change value on $\{t_k\}_{k=0}^{\infty}$ where the sequence $\{t_k\}_{k=0}^{\infty}$ satisfies the range dwell-time condition $T_k := t_{k+1} - t_k \in [T_{min}, T_{max}]$, $0 < T_{min} \leq T_{max} < \infty$, for all $k \in \mathbb{Z}_{\geq 0}$.

The objective of this section is to obtain co-design conditions for the simultaneous design of the state-feedback control gains $K_i^1 \in \mathbb{R}^{m \times n}, K_i^2 \in \mathbb{R}^{m \times m}$, $i = 1, \dots, N$ and the jump scheduling law $\sigma(t)$ that is actuated only at the times in $\{t_k\}_{k=0}^{\infty}$ using only current state-information. In order to solve this problem, we first rewrite the impulsive sampled-data system (1) into an impulsive system with augmented state-space

$$\begin{aligned} \dot{\chi}(t) &= \underbrace{\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}}_{\bar{A}} \chi(t), t \in \mathbb{R}_{\geq 0} \setminus \{t_k\}_{k=0}^{\infty} \\ \chi(t_k^+) &= \underbrace{\begin{bmatrix} J_{\sigma(t_k^+)} & 0 \\ K_{\sigma(t_k^+)}^1 & K_{\sigma(t_k^+)}^2 \end{bmatrix}}_{\bar{J}_{\sigma(t_k^+)}} \chi(t_k), k \in \mathbb{Z}_{\geq 0} \end{aligned} \quad (2)$$

where $\chi(t) := \text{col}(x(t), u(t))$, $\bar{J}_i =: \bar{J}_i^0 + \bar{J}_i^1 K_i$, $K_i := [K_i^1 \ K_i^2]$, $i = 1, \dots, N$, and for which we propose the following *min-jumping rule*

$$\sigma(t_k^+) = \arg \min_{i \in \{1, \dots, N\}} \{ \chi(t_k)^T P_i \chi(t_k) \} \quad (3)$$

where the matrices $P_i \in \mathbb{S}_{> 0}^{n+m}$ have to be designed. This rule is clearly inspired from the continuous min-switching law considered in [3] in the context of switched systems.

The following result states a sufficient condition for the stability of the system (1) controlled with the min-jumping rule (3):

Proposition 1 Let $0 < T_{\min} \leq T_{\max} < \infty$ and assume that there exist some matrices $P_i \in \mathbb{S}_{>0}^{n+m}$, $i = 1, \dots, N$, and a nonnegative matrix $\Pi \in \mathbb{R}^{N \times N}$ verifying $\mathbf{1}_N^T \Pi = \mathbf{1}_N^T$ such that the condition

$$\bar{J}_i^T e^{\bar{A}^T \theta} \left(\sum_{j=1}^N \pi_{ji} P_j \right) e^{\bar{A} \theta} \bar{J}_i - P_i \prec 0 \quad (4)$$

holds for all $i = 1, \dots, N$ and all $\theta \in [T_{\min}, T_{\max}]$.

Then, the system (1) controlled with the rule (3) is asymptotically stable for any sequence $\{t_k\}_{k=0}^\infty$ satisfying the range dwell-time condition $t_{k+1} - t_k \in [T_{\min}, T_{\max}]$.

Proof : The proof of this result is based on the consideration of the linear discrete-time switched system

$$\chi(t_{k+1}) = e^{\bar{A} T_k} \bar{J}_{\sigma(t_k^+)} \chi(t_k) \quad (5)$$

whose stability is equivalent to the original impulsive system. Using now the result in [4] for discrete-time switched systems yields the result. \diamond

The condition (4) is not an LMI because of the products between P_j and π_{ji} but becomes one whenever the π_{ji} 's are chosen a priori. Moreover, when the state-feedback gains K_i are to be computed, then the condition (4) is not appropriate since it cannot be easily turned into a form that is convenient for design purposes. At last, the presence of the uncertain parameter θ at the exponential adds complexity to the overall approach. However, this latter problem has now been extensively studied; see e.g. [2, 9, 12, 15] and subsequent works of the same authors.

Inspired from the results in [2, 11], the following co-design result is obtained:

Theorem 2 Let $0 < T_{\min} \leq T_{\max} < \infty$. Then, the following statements are equivalent:

- (a) There exist some matrices $P_i \in \mathbb{S}_{>0}^{n+m}$, $K_i \in \mathbb{R}^{m \times (n+m)}$, $i = 1, \dots, N$, and a nonnegative matrix $\Pi \in \mathbb{R}^{N \times N}$ verifying $\mathbf{1}_N^T \Pi = \mathbf{1}_N^T$ such that the condition (4) holds for all $i = 1, \dots, N$ and all $\theta \in [T_{\min}, T_{\max}]$.
- (b) There exist some differentiable matrix-valued functions $S_i : [0, T_{\max}] \mapsto \mathbb{S}^{n+m}$, some matrices $P_i \in \mathbb{S}_{>0}^{n+m}$, $K_i \in \mathbb{R}^{m \times (n+m)}$, $i = 1, \dots, N$, a nonnegative matrix $\Pi \in \mathbb{R}^{N \times N}$ verifying $\mathbf{1}_N^T \Pi = \mathbf{1}_N^T$ and a scalar $\varepsilon > 0$ such that the conditions

$$-\dot{S}_i(\tau) + \bar{A}^T S_i(\tau) + S_i(\tau) \bar{A} \preceq 0 \quad (6)$$

$$-P_i + \bar{J}_i^T S_i(\theta) \bar{J}_i + \varepsilon I \preceq 0 \quad (7)$$

and

$$\sum_{j=1}^N \pi_{ji} P_j - S_i(0) \preceq 0 \quad (8)$$

hold for all $i = 1, \dots, N$, all $\tau \in [0, T_{\max}]$ and all $\theta \in [T_{\min}, T_{\max}]$.

- (c) There exist some differentiable matrix-valued functions $\tilde{S}_i : [0, T_{\max}] \mapsto \mathbb{S}^{n+m}$, some matrices $\tilde{P}_i \in \mathbb{S}_{>0}^{n+m}$, $U_i \in \mathbb{R}^{m \times (n+m)}$, $i = 1, \dots, N$, a nonnegative matrix $\Pi \in \mathbb{R}^{N \times N}$ verifying $\mathbf{1}_N^T \Pi = \mathbf{1}_N^T$ and a scalar $\varepsilon > 0$ such that the conditions

$$\dot{\tilde{S}}_i(\tau) + \tilde{S}_i(\tau) \bar{A}^T + \bar{A} \tilde{S}_i(\tau) \preceq 0 \quad (9)$$

$$\begin{bmatrix} -\tilde{P}_i & \star \\ \bar{J}_i^0 \tilde{P}_i + \bar{J}_i^1 U_i & -\tilde{S}_i(\theta) \end{bmatrix} \prec 0 \quad (10)$$

and

$$-\text{diag}\{\tilde{P}_j\} + V_i \tilde{S}_i(0) V_i^T \preceq 0 \quad (11)$$

where $V_i = \text{col}_{j=1}^N \{\pi_{ji}^{1/2} I_{n+m}\}$ hold for all $i = 1, \dots, N$, all $\tau \in [0, T_{\max}]$ and all $\theta \in [T_{\min}, T_{\max}]$.

Moreover, when the conditions of statement (c) hold, then the conditions in Proposition 1 hold with $K_i = U_i \tilde{P}_i^{-1}$ and $P_i = \tilde{P}_i^{-1}$, $i = 1, \dots, N$. As a result, the system (1) with the controller gains $K_i = U_i \tilde{P}_i^{-1}$, $i = 1, \dots, N$, and the rule (3) with $P_i = \tilde{P}_i^{-1}$ is asymptotically stable for any sequence $\{t_k\}_{k=0}^{\infty}$ satisfying the range dwell-time condition $t_{k+1} - t_k \in [T_{\min}, T_{\max}]$.

Proof : The proof of the equivalence between the three statements is adapted from previous works of the author [2, 11, 14] and is only sketched. To prove that (a) implies (b), we assume that the conditions of statement (a) hold and let $S_i^*(\tau) := e^{\bar{A}^T \tau} \left[\sum_{j=1}^N \pi_{ji} P_j \right] e^{\bar{A} \tau}$, then we immediately get that (6) and (8) hold for $S_i = S_i^*$. Evaluating then (7) with $\tilde{S}_i = S_i^*$, we get that is identical to (4), which is satisfied by assumption. The proof is completed.

To prove the converse, first observe that by integrating (6) over $[0, \theta]$, we get that

$$e^{\bar{A}^T \theta} S_i(0) e^{\bar{A} \theta} - S_i(\theta) \preceq 0$$

which together with (8) implies that

$$e^{\bar{A}^T \theta} \left[\sum_{j=1}^N \pi_{ji} P_j \right] e^{\bar{A} \theta} - S_i(\theta) \preceq 0.$$

By pre- and post-multiplying the above inequality by \bar{J}_i^T and \bar{J}_i , respectively, and using (7), we finally obtain (4). The proof is completed.

The proof that (b) is equivalent to (c) follows from the changes of variables $U_i = K_i \tilde{P}_i$, $\tilde{P}_i = P_i^{-1}$, $\tilde{S}_i(\tau) = S_i(\tau)^{-1}$ and Schur complements. \diamond

The conditions stated in statement (c) are more convenient to consider than those stated in Proposition 1 since the uncertainty θ is not at the exponential anymore and the conditions are convex in the decision matrices P_i 's and K_i 's whenever the matrix Π is chosen a priori. More specifically, these conditions are convex infinite-dimensional LMI conditions that can be solved in an efficient way using discretization methods [12, 14, 16] or sum of squares methods [2, 11, 13, 14], which can be both shown to be asymptotically exact. On the other hand, when Π has to be designed, the problem becomes nonlinear and may be difficult to solve; see [3, 4] for some discussion on how to potentially solve such a problem.

3 Extension to switched impulsive systems

Let us now consider the following class of switched impulsive systems with multiple jump maps and sampled-data state-feedback:

$$\begin{aligned} \dot{x}(t) &= A_{\sigma(t)} x(t) + B_{\sigma(t)} u(t), \quad t \in \mathbb{R}_{\geq 0} \setminus \{t_k\}_{k=0}^{\infty} \\ u(t) &= K_{\sigma(t_k^+), \sigma(t_k)}^1 x(t_k) \\ &\quad + K_{\sigma(t_k^+), \sigma(t_k)}^2 u(t_k), \quad t \in (t_k, t_{k+1}] \\ x(t_k^+) &= J_{\sigma(t_k^+), \sigma(t_k)} x(t_k), \quad k \in \mathbb{Z}_{\geq 0} \end{aligned} \tag{12}$$

where $x, x_0 \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state of the system, the initial condition and the control input, respectively. As before, the switching signal $\sigma : \mathbb{R}_{\geq 0} \mapsto \{1, \dots, N\}$ is assumed to be piecewise constant and to only change values on $\{t_k\}_{k=0}^{\infty}$.

The above system can be equivalently represented by the impulsive-switched system

$$\begin{aligned} \dot{\chi}(t) &= \bar{A}_{\sigma(t)} \chi(t), \quad t \in \mathbb{R}_{\geq 0} \setminus \{t_k\}_{k=0}^{\infty} \\ \chi(t_k^+) &= \bar{J}_{\sigma(t_k^+), \sigma(t_k)} \chi(t_k), \quad k \in \mathbb{Z}_{\geq 0} \end{aligned} \tag{13}$$

with $\chi = \text{col}(x, u)$, $K_{j,i} := [K_{j,i}^1 \ K_{j,i}^2]$,

$$\bar{A}_i = \begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix}, \quad \bar{J}_{j,i} = \begin{bmatrix} J_{j,i} & 0 \\ K_{j,i}^1 & K_{j,i}^2 \end{bmatrix} =: \bar{J}_{j,i}^0 + \bar{J}_{j,i}^1 K_{j,i} \tag{14}$$

for $i, j = 1, \dots, N$ and for which we propose the jump rule:

$$\sigma(t_k^+) = \arg \min_{j \in \{1, \dots, N\}} \left\{ \chi(t_k)^T \bar{J}_{j, \sigma(t_k)}^T P_j \bar{J}_{j, \sigma(t_k)} \chi(t_k) \right\}. \quad (15)$$

The following result states a sufficient condition for the stability of the system (12) controlled with the min-jumping rule (15):

Proposition 3 *Let $0 < T_{\min} \leq T_{\max} < \infty$ and assume that there exist some matrices $P_i \in \mathbb{S}_{>0}^{n+m}$ and a nonnegative matrix $\Pi \in \mathbb{R}^{N \times N}$ verifying $\mathbf{1}_N^T \Pi = \mathbf{1}_N^T$ such that the condition*

$$e^{\bar{A}_i^T \theta} \left(\sum_{j=1}^N \pi_{ji} \bar{J}_{j,i}^T P_j \bar{J}_{j,i} \right) e^{\bar{A}_i \theta} - P_i \prec 0 \quad (16)$$

holds for all $i = 1, \dots, N$ and all $\theta \in [T_{\min}, T_{\max}]$.

Then, the system (12) controlled with the min-jumping rule (15) is asymptotically stable for any sequence $\{t_k\}_{k=0}^\infty$ satisfying the range dwell-time condition $t_{k+1} - t_k \in [T_{\min}, T_{\max}]$.

Proof : The proof consists of an extension of a proof in [4] and is given below for completeness. Let $\sigma(t_k) = \sigma(t_{k-1}^+) = i$ and assume that the conditions of Proposition 3 hold. Define then the function

$$\begin{aligned} V(k) &:= \min_{j \in \{1, \dots, N\}} \{ \chi(t_k^+)^T P_j \chi(t_k^+) \} \\ &= \min_{j \in \{1, \dots, N\}} \{ \chi(t_k)^T \bar{J}_{j,i}^T P_j \bar{J}_{j,i} \chi(t_k) \} \end{aligned} \quad (17)$$

which is consistent with the rule (3). Then, we have that

$$\begin{aligned} V(k) &= \min_{\substack{\lambda \geq 0 \\ \mathbf{1}_N^T \lambda = 1}} \left\{ \chi(t_k)^T \left(\sum_{j=1}^N \lambda_j \bar{J}_{j,i}^T P_j \bar{J}_{j,i} \right) \chi(t_k) \right\} \\ &\leq \chi(t_{k-1}^+)^T e^{A_i^T T_k} \left(\sum_{j=1}^N \pi_{ji} \bar{J}_{j,i}^T P_j \bar{J}_{j,i} \right) e^{A_i T_k} \chi(t_{k-1}^+) \\ &< \chi(t_{k-1}^+)^T P_i \chi(t_{k-1}^+) = V(k-1) \end{aligned} \quad (18)$$

provided that $T_k \in [T_{\min}, T_{\max}]$. So, V is a discrete-time Lyapunov function for the system

$$\chi(t_k^+) = \bar{J}_{\sigma(t_k^+), \sigma(t_k)} e^{\bar{A}_{\sigma(t_k)} T_k} \chi(t_{k-1}^+) \quad (19)$$

controlled with the rule (15). Since the stability of the above discrete-time system is equivalent to that of (12) under the min-jumping rule (15) then we can conclude that the system (12) controlled with the min-jumping rule (15) is asymptotically stable for any sequence $\{t_k\}_{k=0}^\infty$ satisfying the range dwell-time condition $t_{k+1} - t_k \in [T_{\min}, T_{\max}]$. This completes the proof. \diamond

As for Proposition 1, the conditions in Proposition 3 are not directly tractable and need to be reformulated in this regard. This is stated in the following result:

Theorem 4 *Let $0 < T_{\min} \leq T_{\max} < \infty$. Then, the following statements are equivalent:*

- (a) *There exist some matrices $P_i \in \mathbb{S}_{>0}^{n+m}$, $K_{ij} \in \mathbb{R}^{m \times (n+m)}$, $i, j = 1, \dots, N$, and a nonnegative matrix $\Pi \in \mathbb{R}^{N \times N}$ verifying $\mathbf{1}_N^T \Pi = \mathbf{1}_N^T$ such that the conditions in Proposition 3 hold.*

- (b) There exist some differentiable matrix-valued functions $S_i : [0, T_{max}] \mapsto \mathbb{S}^{n+m}$, some matrices $P_i \in \mathbb{S}_{>0}^{n+m}$, $K_i \in \mathbb{R}^{m \times (n+m)}$, $i = 1, \dots, N$, a nonnegative matrix $\Pi \in \mathbb{R}^{N \times N}$ verifying $\mathbf{1}_N^T \Pi = \mathbf{1}_N^T$ and a scalar $\varepsilon > 0$ such that the conditions

$$-\dot{S}_i(\tau) + \bar{A}_i^T S_i(\tau) + S_i(\tau) \bar{A}_i \preceq 0 \quad (20)$$

$$-P_i + S_i(\theta) + \varepsilon I \preceq 0 \quad (21)$$

and

$$\sum_{j=1}^N \pi_{ji} \bar{J}_{j,i}^T P_j \bar{J}_{j,i} - S_i(0) \preceq 0 \quad (22)$$

hold for all $i = 1, \dots, N$, all $\tau \in [0, T_{max}]$ and all $\theta \in [T_{min}, T_{max}]$.

- (c) There exist some differentiable matrix-valued functions $\tilde{S}_i : [0, T_{max}] \mapsto \mathbb{S}^{n+m}$, some matrices $\tilde{P}_i \in \mathbb{S}_{>0}^{n+m}$, $U_{i,j} \in \mathbb{R}^{m \times (n+m)}$, $i, j = 1, \dots, N$, a nonnegative matrix $\Pi \in \mathbb{R}^{N \times N}$ verifying $\mathbf{1}_N^T \Pi = \mathbf{1}_N^T$ and a scalar $\varepsilon > 0$ such that the conditions

$$\dot{\tilde{S}}_i(\tau) + \tilde{S}_i(\tau) \bar{A}_i^T + \bar{A}_i \tilde{S}_i(\tau) \preceq 0 \quad (23)$$

$$\tilde{P}_i - \tilde{S}_i(\theta) + \varepsilon I \preceq 0 \quad (24)$$

and

$$\begin{bmatrix} -\tilde{S}_i(0) & V_i^T \\ V_i & -\text{diag}_{j=1}^N \{\tilde{P}_j\} \end{bmatrix} \preceq 0 \quad (25)$$

hold for all $i = 1, \dots, N$ and all $\theta \in [T_{min}, T_{max}]$ where $V_i = \text{col}_{j=1}^N \{\pi_{ji}^{1/2} [\bar{J}_{j,i}^0 \tilde{S}_i(0) + \bar{J}_{j,i}^1 U_{j,i}]\}$.

Moreover, when the conditions of statement (c) hold, then the conditions of Proposition 3 hold with $K_{j,i} = U_{j,i} \tilde{S}_i(0)^{-1}$, $i, j = 1, \dots, N$, $i \neq j$, and $P_i = \tilde{P}_i^{-1}$, $i = 1, \dots, N$. As a result, the system (12) with controller gains $K_{j,i} = U_{j,i} \tilde{S}_i(0)^{-1}$ and min-jumping rule (15) is asymptotically stable for any sequence $\{t_k\}_{k=0}^\infty$ satisfying the range dwell-time condition $t_{k+1} - t_k \in [T_{min}, T_{max}]$.

Proof : This result can be proven in the same way as Theorem 2. \diamond

4 Examples

We provide several illustrative examples. The infinite-dimensional conditions stated in the main results of the paper are checked using the package SOSTOOLS [17] and the semidefinite programming solver SeDuMi [18]. Examples of sum of squares programs can be found in [2, 11, 14].

4.1 Example 1

Let us consider the system (1) with the matrices

$$A = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (26)$$

and we define

$$\bar{J}_1 = \left[\begin{array}{c|c} I_2 & 0 \\ \hline K^1 & K^2 \end{array} \right] \quad \text{and} \quad \bar{J}_2 = \left[\begin{array}{cc|c} 0.7 & 0 & 0 \\ 0 & 1.1 & 0 \\ \hline 0 & 0 & 1 \end{array} \right]. \quad (27)$$

From the definition of the system, we can see that a game needs to be played between the two jump matrices. Indeed, the first one can stabilize the second state of the system but leaves the dynamics of the first one

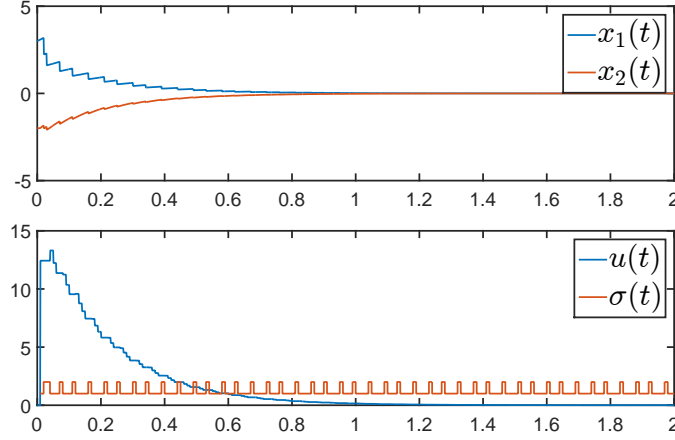


Figure 1: Simulation results for the system (1)-(26)-(27).

unchanged. On the other hand, the second jump matrix can stabilize the first state but destabilizes the second one. Applying then the conditions of Theorem 2(c) with $T_{min} = 10\text{ms}$, $T_{max} = 50\text{ms}$, $\pi_{12} = \pi_{21} = 0.1$ with polynomials of order 2 in the associated sum of squares conditions yields the matrices of the min-jumping rule given by

$$P_1 = \begin{bmatrix} 0.1184 & 0.0184 & 0.0023 \\ 0.0184 & 0.5032 & 0.0183 \\ 0.0023 & 0.0183 & 0.0027 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.0866 & 0.0877 & 0.0108 \\ 0.0877 & 1.3107 & 0.1124 \\ 0.0108 & 0.1124 & 0.0142 \end{bmatrix}$$

and the following controller gain

$$[K^1 \mid K^2] = [-0.9622 \quad -7.7351 \mid -0.0260]. \quad (28)$$

Simulation results are depicted in Fig. 1 where we can see that the co-designed sampled-data jump rule and state-feedback control law are effectively able to stabilize the open-loop unstable system.

4.2 Example 2

We consider here the system (1) with the matrices

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}, \quad J_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0.8 \end{bmatrix} \quad \text{and} \quad J_2 = \begin{bmatrix} 0.7 & 0 \\ 0 & 1 \end{bmatrix} \quad (29)$$

where we can see that, as before, a game needs to be played between the two jump matrices. Applying the conditions of Theorem 2(c) (adapted to the case where no state-feedback controller has to be designed) with $\pi_{11} = \pi_{22} = 0.9$, $T_{min} = T_{max} = 0.02$ yields the matrices

$$P_1 = \begin{bmatrix} 25.5386 & 6.3780 \\ 6.3780 & 6.6746 \end{bmatrix} \quad \text{and} \quad P_2 = \begin{bmatrix} 2.8886 & 2.8549 \\ 2.8549 & 20.6927 \end{bmatrix}. \quad (30)$$

The simulation results in Fig. 2 illustrate the ability of the min-switching rule to efficiently stabilize the system.

4.3 Example 3

Let us consider the system (12) with the matrices

$$A_1 = A_2 = \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 6 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}. \quad (31)$$

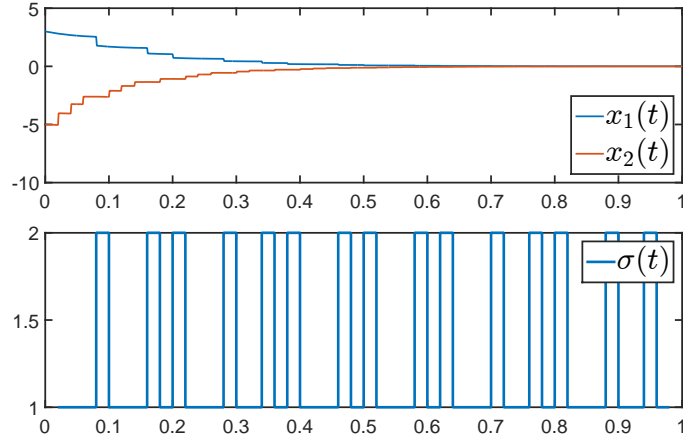


Figure 2: Simulation results for the system (1)-(29).

This system represents a system for which only one actuator can be updated at a time while the other maintains its previous control input. The first one can act on both states (i.e. the pair (A, B_1) is controllable) whereas the second one can only act on the second state. Applying the conditions of Theorem 4(c) with $\pi_{11} = \pi_{22} = 0.1$, $T_{min} = 10\text{ms}$, $T_{max} = 50\text{ms}$, with polynomials of order 2 in the associated sum of squares conditions yields the matrices

$$P_1 = 10 \begin{bmatrix} 0.63 & -0.06 & -2.14 & -0.11 \\ -0.06 & 3.51 & 0.03 & -7.64 \\ -2.14 & 0.03 & 7.45 & 0.59 \\ -0.11 & -7.64 & 0.59 & 203.99 \end{bmatrix}$$

$$P_2 = 10 \begin{bmatrix} 0.43 & -0.05 & -1.53 & -0.08 \\ -0.05 & 4.03 & 0.05 & -8.53 \\ -1.53 & 0.05 & 191.30 & 0.61 \\ -0.08 & -8.53 & 0.61 & 23.52 \end{bmatrix}$$

together with

$$\begin{aligned} K_{1,1} &= [-3.4332 \quad -0.0457 \quad -0.0061 \quad -0.0003], \\ K_{1,2} &= [-3.4160 \quad -0.0516 \quad -0.0001 \quad -0.0033], \\ K_{2,1} &= [-0.4073 \quad -2.1272 \quad 0.0076 \quad -0.0004], \\ K_{2,2} &= [-0.4323 \quad -2.1198 \quad 0.0014 \quad 0.0043]. \end{aligned}$$

The simulation results depicted in Fig. 3 demonstrate the stabilization effect of the proposed co-design approach.

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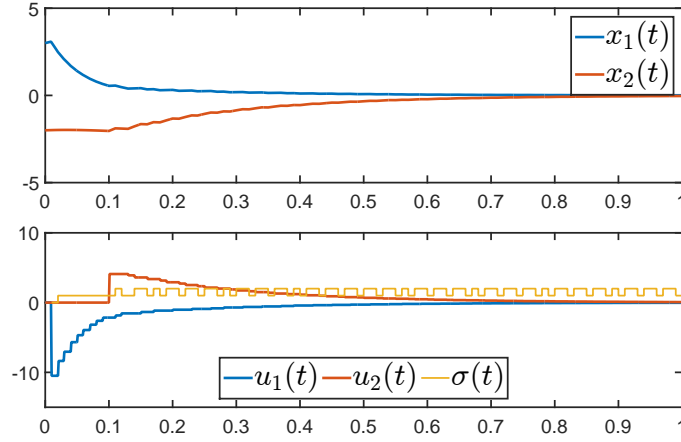


Figure 3: Simulation results for the system (12)-(31).

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